Séminaire N. Bourbaki

SAMEDI 28 MARS 2026

Institut Henri Poincaré (amphithéatre Charles Hermite)
11 rue Pierre et Marie Curie, 75005 Paris

10h00 Abed BOUNEMOURA
Instabilité dans le problème planétaire

Le problème planétaire à $1+n$ corps est un système d'équations différentielles modélisant l'évolution du système solaire, à savoir le mouvement de n corps (« planètes ») en interaction gravitationnelle autour d'un corps plus massif (« Soleil »). Les mathématiciens et astronomes ont cru pendant longtemps à sa stabilité : les trajectoires elliptiques (képlériennes), présentes lorsque les planètes n'interagissent pas entre elles, ne sont que légèrement déformées lorsque l'on restaure cette interaction (« théorème » de stabilité de Laplace–Lagrange). Mais depuis les travaux de Poincaré et d'Arnold, on s'attend au contraire à de l'instabilité, bien que cela ait résisté aux efforts des mathématiciens. Le but de l'exposé est d'expliquer un résultat de Clarke, Fejoz et Guardia qui montre que pour le problème à $1+n=4$ corps (ou plus), il existe des mouvements dont le demi-grand axe d'un des corps a une variation aussi grande que l'on veut. Ceci montre que la conclusion du théorème de stabilité de Laplace–Lagrange n'est pas valide pour le problème planétaire, et résout une conjecture d'Arnold.

11h30 Larry GUTH
The Kakeya conjecture in $\mathbb{R}^3$, *after Hong Wang and Joshua Zahl*

The Kakeya conjecture is a question about how thin tubes can overlap with each other in Euclidean space. It connects to problems in Fourier analysis and PDE, such as understanding L^p type estimates for solutions of the wave equation. This connection has prompted a lot of work on the problem in the harmonic analysis community.

The 2-dimensional case of the Kakeya conjecture has a 1-page proof and has been known since the 1970s. Recently, Wang and Zahl proved the 3-dimensional case of the Kakeya conjecture. Their work builds on contributions of many people, including Bourgain, Wolff, Katz, Laba, Tao, Orponen, and Shmerkin.

There are interesting examples related to algebraic geometry which show that some cousins of the Kakeya conjecture are false. These counterexamples are also important clues for understanding the problem. In the proof of Kakeya, we imagine a hypothetical counterexample, we prove that it would have to have a great deal of algebraic structure, and we finally show that it cannot exist at all. A hypothetical counterexample to the Kakeya conjecture is just a set of thin tubes in 3-dimensional space which overlap each other a lot. We will try to describe how to prove that such a set has algebraic structure.

14h30 Jordan ELLENBERG
Recent Progress around Cohen–Lenstra Heuristics

In 1983, Henri Cohen and Hendrik Lenstra proposed a conjecture about the distribution of the N-torsion of the class group of a random quadratic field, supported by what was at the time a large amount of computational evidence. The Cohen–Lenstra heuristics, which are still almost entirely unproven, have become one of the central foundational problems in arithmetic statistics. Recent years have seen a rapidly accelerated pace of development in Cohen-Lenstra problems. I will give a tour of these developments, including the work of Wood and her collaborators developing a fully fleshed out roster of generalized Cohen–Lenstra conjectures, with support from topology; Smith's theorems proving the Cohen–Lenstra conjectures for the 2-primary part of the class group, as part of more general theorems about Selmer groups in quadratic twists, leading to a resolution of the minimalist conjecture for elliptic curves; and recent work by Koymans and Pagano in the 2-primary case, expanding on Smith's work and proving Stevenhagen's conjecture on the negative Pell equation.

16h00 Peter SCHOLZE
Geometric Langlands, *after Gaitsgory, Raskin, ...*

At its core, the Langlands program seeks to give a description of the vector space of automorphic forms. This is a space of functions on a locally symmetric space X/Γ, where $\Gamma = G(\mathbb{Z}) \subset G(\mathbb{R})$ is an arithmetic group, equipped with its Hecke symmetries. One seeks a "dual" description, in terms of L-parameters, which are roughly representations of the absolute Galois group with values in the Langlands dual group.

Similar conjectures exist in the function-field case; in this setting, the quotient X/Γ can be interpreted as the $\mathbb{F}_p$-points of the stack Bun_G of G-bundles on a curve C over $\mathbb{F}_p$. Gaitsgory–Raskin, partly in large collaborations, proved a fine version of Langlands' conjecture in this setting, in the everywhere unramified case. Notably, they define a vector space on the Galois side, as global sections of a sheaf on the moduli stack of L-parameters, and relate this to the space of automorphic forms, compatibly with the Hecke symmetries.

Their proof is a culmination of many decades of work in the geometric Langlands program, and consists of two key steps: 1) The space of automorphic forms is the trace of Frobenius on a category of ℓ-adic sheaves on Bun_G. 2) This category of ℓ-adic sheaves on Bun_G is equivalent to a category of coherent sheaves on the stack of L-parameters. The starting point for 2) is the observation that this conjecture makes sense for curves over any field, and any sheaf theory, and is known as the geometric Langlands equivalence. The proof starts over the complex numbers, and uses D-modules.