Séminaire N. Bourbaki

SAMEDI 1 AVRIL 2023

Institut Henri Poincaré (amphi. Hermite)
11 rue Pierre et Marie Curie, 75005 Paris

10h Jonathan HICKMAN
Pointwise convergence for the Schrödinger equation,
after Xiumin Du and Ruixiang Zhang

For an initial datum $f \in L^2(\mathbf{R}^n)$, consider the linear Schrödinger equation

$$\begin{cases} iu_t - \Delta_x u = 0, \\ u(x,0) = f(x) \end{cases} \qquad (x,t) \in \mathbf{R}^n \times \mathbf{R}.$$

In 1980, Carleson asked which additional conditions on f guarantee

$$(\star) \qquad \lim_{t \to 0} u(x,t) = f(x) \qquad \text{for almost every } x \in \mathbf{R}^n.$$

More precisely, what is the minimal Sobolev regularity index s such that ($\star$) holds whenever $f \in H^s(\mathbf{R}^n)$?

Whilst the $n = 1$ case was fully understood by the early 1980s, in higher dimensions the situation is much more nuanced. Nevertheless, a recent series of dramatic developments brought about an almost complete resolution of the problem. First Bourgain (2016) produced a subtle counterexample demonstrating that pointwise convergence can fail for certain $f \in H^s(\mathbf{R}^n)$ with $s < \frac{n}{2(n+1)}$. Complementing this, convergence was then shown to hold for $s > \frac{n}{2(n+1)}$ when $n = 2$ in a landmark paper of Du, Guth and Li (2017) and later in all dimensions in equally important work of Du and Zhang (2019).

This seminar will explore the positive result of Du and Zhang (2019). The argument combines sophisticated modern machinery from harmonic analysis such as the multilinear Strichartz estimates of Bennett, Carbery and Tao (2006) and the ℓ^2 decoupling theory of Bourgain and Demeter (2015). However, equally important are a variety of elementary guiding principles, rooted in Fourier analysis, which govern the behaviour of solutions to the Schrödinger equation. The talk will focus on these basic Fourier analytic principles, building intuition and presenting a powerful toolbox for tackling problems in modern PDE and harmonic analysis.

11h30 Matteo VIALE

Strong forcing axioms and the continuum problem,
following Aspéro's and Schindler's proof that $\mathbf{MM}^{++}$ *implies Woodin's Axiom* $(*)$

A topological approach to forcing axioms considers them as strong forms of the Baire category theorem; an algebraic approach describes certain properties of "algebraic closure" for the universe of sets that can be derived from them. Our goal is to show how the theorem of Aspéro and Schindler links the geometric and algebraic points of view. Drawing on Gödel's program, we connect these mathematical results to the philosophical debate on what could constitute a viable solution of the continuum problem.

14h30 Clara LÖH

Exponential growth rates in hyperbolic groups,
after Koji Fujiwara and Zlil Sela

A classical result of Jørgensen and Thurston shows that the set of volumes of finite volume complete hyperbolic 3-manifolds is a well-ordered subset of the real numbers of order type ω^ω; moreover, they showed that each volume can only be attained by finitely many isometry types of hyperbolic 3-manifolds.

Fujiwara and Sela established a group-theoretic companion of this result: If Γ is a non-elementary hyperbolic group, then the set of exponential growth rates of Γ is well-ordered, the order type is at least ω^ω, and each growth rate can only be attained by finitely many finite generating sets (up to automorphisms).

In this talk, I will outline this work of Fujiwara and Sela and discuss related results.

16h Étienne GHYS

Le groupe des homéomorphismes de la sphère de dimension 2 **qui respectent l'aire et l'orientation n'est pas un groupe simple**,
d'après D. Cristofaro-Gardiner, V. Humilière et S. Seyfaddini

Depuis la fin des années 1970, on sait que la composante neutre du groupe des difféomorphismes à support compact d'une variété connexe est un groupe simple. Dans le cas difféomorphismes qui préservent une forme volume ou une forme symplectique, on dispose d'un résultat analogue : il a alors un sous-groupe « évident » qui est simple. Pour les homéomorphismes qui respectent le volume, la situation est comprise lorsque la dimension est supérieure ou égale à 3. Le cas des surfaces, et tout particulièrement de la sphère de dimension 2, a résisté à de nombreux efforts depuis une quarantaine d'années. Le théorème de D. Cristofaro-Gardiner, V. Humilière et S. Seyfaddini est une surprise : le groupe des homéomorphismes de la sphère de dimension 2 qui respectent l'aire et l'orientation n'est *pas* un groupe simple. La démonstration est un tour de force et fait largement usage de l'homologie de Floer périodique. J'essaierai de présenter le contexte ainsi que les grandes lignes de ce beau résultat.