Séminaire N. Bourbaki

SAMEDI 27 NOVEMBRE 2021

Institut Henri Poincaré (amphi. Hermite)
11 rue Pierre et Marie Curie, 75005 Paris

11h00 Sylvy ANSCOMBE
Shelah's Conjecture and Johnson's Theorem

The "Shelah Conjecture" proposes a description of fields whose first-order theories are without the Independence Property (IP): they are finite, separably closed, real closed, or admit a non-trivial henselian valuation. One of the most prominent dividing lines in the contemporary model-theoretic universe, IP holds in a theory if there is a formula that can define arbitrary subsets of arbitrarily large finite sets. In 2020, Johnson gave a proof of the conjecture in an important case; namely, the case of dp-finite (roughly: finite dimensional) theories of fields. Combined with a result of Halevi–Hasson–Jahnke, Johnson's Theorem completely classifies the dp-finite theories of fields.
We will explain this classification, describe some ingredients of the proof, and explore how Johnson's Theorem and the Shelah Conjecture fit into the bigger picture.

14h30 Menny AKA
Joinings classification and applications, *after Einsiedler and Lindenstrauss*

This talk surveys the classification of joinings of higher-rank torus actions on S-arithmetic quotients of semisimple or perfect algebraic groups and some of its applications. This classification was proved by Einsiedler and Lindenstrauss (Duke Mathematical Journal 2007, Publications mathématiques de l'IHÉS, 2019). It establishes that ergodic joinings must be algebraic, and in particular that such torus actions in many cases must be disjoint, that is, they admit only the trivial joining which is the product of the Haar measures on each of the factors.
Their proof is based on entropy methods, developed by Einsiedler, Katok, Lindenstrauss and Spatzier. We will describe these methods and give some ideas on how they fit into the scheme of their proof. Specifically, we will explain how to prove disjointness when the associated algebraic groups have a different root structure. This already allows for some applications, which will be presented at the end of the talk.

16h00 Uli WAGNER
High-Dimensional Expanders,
after Gromov, Kaufman, Kazhdan, Lubotzky, and others

Expander graphs (sparse but highly connected graphs) have, since their inception, been the source of deep links between Mathematics and Computer Science as well as applications to other areas. In recent years, a fascinating theory of high-dimensional expanders has begun to emerge, which is still in a formative stage but has nonetheless already lead to a number of striking results. Unlike for graphs, in higher dimensions there is a rich array of non-equivalent notions of expansion (coboundary expansion, cosystolic expansion, topological expansion, spectral expansion, etc.), with differents strengths and applications. In this talk, we will survey this landscape of high-dimensional expansion, with a focus on two main results. First, we will present Gromov's Topological Overlap Theorem, which asserts that coboundary expansion (a quantitative version of vanishing mod 2 cohomology) implies topological expansion (roughly, the property that for every map from a simplicial complex to a manifold of the same dimension, the images of a positive fraction of the simplices have a point in common). Second, we will outline a construction of bounded degree 2-dimensional topological expanders, due to Kaufman, Kazhdan, and Lubotzky.