ABSTRACTS

Gil KALAI — *Designs exist!* [*after Peter Keevash*]

One of the central and oldest problems in combinatorics is: *Can you find a collection S of q-subsets from an n-element set X so that every r-subset of X is included in precisely t sets in the collection?* A collection S of this kind is called a design of parameters (n, q, r, t), a special interest is the case $t = 1$, and in this case S is called a Steiner system. It was conjectured that a design of parameters (n, q, r, t) exists for every q, r and t if n is sufficiently large and is admissible, namely if some necessary simple divisibility conditions are satisfied. Until recently the known constructions came very far from this.

Here is a brief history of the problem: The existence of designs and Steiner systems was asked by Plücker (1835), Kirkman (1846) and Steiner (1853). For $r = 2$ which was of special interest, Richard Wilson (1972–1975) proved their existence for large enough admissible values of n. Rödl (1985) proved the existence of approximate objects (the property holds for $(1 - o(1))$ r-subsets of X), thus answering a conjecture by Erdös and Hanani. Teirlink (1987) proved their existence for infinitely many values of n when r and q are arbitrary and t is a certain large number depending on q and r but not on n. (His construction also does not have repeated blocks.) Keevash (2014) proved the existence of Steiner systems for all but finitely many admissible values of n for every q and r. He uses a new method referred to as Randomized Algebraic Constructions.

In the lecture I will describe the problem and its history and will try to explain some ingredients in the methods by Wilson, Rödl and Keevash.

Alexander MERKURJEV — *Essential dimension*

Essential dimension of an algebraic object is the smallest number of algebraically independent parameters required to define the object. This notion was introduced by J. Buhler and Z. Reichstein in 1997. The relation to different parts of algebra such as algebraic geometry, Galois cohomology and representation theory will be discussed.

Sophie MOREL — *Construction of torsion Galois representations* [*after Peter Scholze*]

Let X be a 3-dimensional hyperbolic variety which is a quotient of hyperbolic 3-space by an "arithmetic" group of isometries. The Langlands program predicts that the singular cohomology of X with coefficients in $\mathbf{Z}/n\mathbf{Z}$ should have a natural action of the absolute Galois group of a number field, even though X is not an algebraic variety. The idea is to realize the torsion cohomology of X as parts of the torsion cohomology of another locally symmetric space that happens to be a Shimura variety, hence an algebraic variety defined over a number field. This idea has been succesfully implemented by Harris-Lan-Taylor-Thorne, Scholze and Boxer (independently and in chronological order); their articles actually treat a much more general case, that of the locally symmetric spaces of a general linear group over a totally real or CM number field. We will try to explain Scholze's approach.

Jean-Marc SCHLENKER — *Flat Lorentzian manifolds as limits of anti-de Sitter manifolds* [*after Danciger, Guéritaud and Kassel*]

Margulis space-times are quotients of the 3-dimensional Minkowski space by (non-abelian) free groups acting properly discontinuously. Goldman, Labourie and Margulis have shown that they are determined by a convex co-compact hyperbolic surface S along with a first-order deformation of the metric which uniformly decreases the lengths of closed geodesics. Danciger, Guéritaud and Kassel show that those space-times are principal $\mathbb{R}$-bundles over S with time-like geodesics as fibers, that they are homeomorphic to the interior of a handlebody, and that they admit a fundamental domain bounded by crooked planes. To obtain those results they show that those Margulis space-times are "infinitesimal" versions of 3-dimensional anti-de Sitter manifolds, and are lead to introduce a new parameterization of the space of deformations of a hyperbolic surface that increase the lengths of all closed geodesics.